

Cranking in isospace

SEBASTIAN GŁOWACZ,¹ WOJCIECH SATULA,^{1,2,3,4,*} and RAMON A. WYSS^{2,†}

¹*Institute of Theoretical Physics, University of Warsaw,
ul. Hoża 69, PL-00 681 Warsaw, Poland*

²*KTH (Royal Institute of Technology),
AlbaNova University Centre, 106 91 Stockholm*

³*Department of Physics, University of Tennessee, Knoxville, TN37996, USA*

⁴*Joint Institute for Heavy Ion Research,*

P.O. Box 2008, MS6374, Oak Ridge, TN37831, USA

(Dated: January 28, 2003)

Abstract

The response of isovector and isoscalar pairing to generalized rotation in isospace is studied. Analytical expressions for non-selfconsistent solutions for different limiting cases of the model are derived. In particular, the connections between gauge relations among pairing gaps and the position of the isocranking axis are investigated in $N=Z$ nuclei. The two domains of collective and non-collective rotation in space are generalized to isospace. The amplitudes for pair-transfer of $T = 0$ and $T = 1$ pairs are also calculated. It is shown that the structure of the $T = 0$ state in odd-odd nuclei prevents any enhancement of pairing transfer also in the presence of strong $T = 0$ correlations. The energy differences of the $T = 0$ and $T = 1$ excitations in odd-odd nuclei are qualitatively reproduced by Total Routhian Surface calculations.

PACS numbers: 21.30.Fe, 21.60.Jz

*Electronic address: satula@fuw.edu.pl

†Electronic address: wyss@kth.se

I. INTRODUCTION

The event of detailed experimental studies of heavy nuclei along the $N=Z$ line has resulted in a revival of theoretical investigations related to the properties of proton-neutron (pn) pairing correlations [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. In $N=Z$ nuclei one expects short range correlations of pn type to be of importance due to the four fold degeneracy of the nuclear wave functions. Of particular interest is the role played by $t=0$ pairing correlations. It is still an issue of debate to what extent the correlations in this channel of the pairing force are characterized by a static gap similar to that of the well established $t=1$ seniority force.

The aim of our work is to devise a mean field model in which we can describe consistently excitations in real- and iso-space on the same footing. In a serie of papers we have shown that the isobaric analogue states as well as ground state masses along the $N=Z$ line form an unique probe to $t=0$ pairing correlations [2, 13, 14, 15, 16]. Within the mean field approach, the energy of isobaric analogue states can be described by means of the isocranking approximation, analogous to the rotational excitations in real space [14, 17]. The associated broken symmetry is the deformation of the pairing field. The direction of the pairing field vector is in turn intimately linked to the direction of the cranking axis in isospace. Hereafter (Sec. III and IV) we derive analytical expressions to elucidate these relations which, in fact, determine the regimes of collective and non-collective rotations in isospace. These expressions and conclusions may be of potential use in other, yet unknown, double-phase paired systems that can be described by means of an external cranking-type hamiltonian.

The collectivity of pairing correlations can be accessed by means of pair transfer [18]. In the analysis of nuclei in the vicinity of the $N=Z$ line, it was concluded that indeed, the $t=1$, pn -pairing exhibits collectivity [19]. From this analysis, it was suggested that there is little evidence of $t=0$ collectivity [20]. Let us point out that similar conclusions concerning the role of $t=0$ pairing has been drawn by Berkeley group [21] based on the analysis of excitation energy spectra in $N=Z$ nuclei. However, to draw definite conclusions a detailed understanding of the structure of both $T = 0$ and $T = 1$ ground states in the odd-odd (o-o) nuclei as well as the structure even-even vacuum are necessary. There are empirical arguments based on isobaric symmetry as well as theoretical arguments based essentially on time-reversal symmetry breaking, which indicate that the structure of the $T = 1$ and

$T = 0$ states in o-o $N=Z$ nuclei is entirely different [15]. The simplest scenario, within the BCS theory, consistent with the data can be reached by assuming that the wave function of the $T = 0$ state in o-o nuclei is a two quasi-particle excitation (2QP), whereas the $T = 0$ ground state in even-even (e-e) nuclei and the lowest $T = 1$ state in o-o nuclei are both local quasiparticle vacua i.e. 0QP states [15]. Within this interpretation the transfer of a $T = 0$ deuteron-like pair will always be strongly quenched, irrespectively of the strength of the $T = 0$ correlations, see Sec. VI. Hence, pair transfer may not necessarily be a good indicator for the strength of $T = 0$ pairing correlations.

The outline of the paper is the following: In Sec. III we derive analytical solutions for the isocranked model with $\mathbf{t}=1$ pairing only. In Sec. IV we extend these solutions to include also $\mathbf{t}=0$ pairing. In Sec. V we discuss the role of the isospin symmetry breaking mechanism due to number projection and its influence on the Wigner energy. In Sec. VI we discuss pair transfer from $T = 0$ and $T = 1$ states in $N=Z$ o-o nuclei. In Sec. VII we investigate the influence $\mathbf{t}=0$ pairing on nuclear deformation by performing Total Routhian Surface (TRS) type calculations for $T = 0$ and $T = 1$ states in $N=Z$ o-o nuclei. In Sec. VIII we discuss briefly the shortcomings of our model due to the lack of the particle-hole isovector field. We summarize and conclude the paper in Sec. IX.

II. GAUGE INVARIANCE PROPERTIES OF PROTON-NEUTRON COUPLED HFB EQUATIONS

The HFB (BCS) equations:

$$\begin{pmatrix} \mathbf{h} & \Delta \\ -\Delta^* & -\mathbf{h}^* \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} U \\ V \end{pmatrix} E \quad (1)$$

are invariant under the transformation:

$$V \longrightarrow e^{i\phi} V \quad (2)$$

which we call later global gauge invariance transformation (GGI) since it requires both proton $V_p \rightarrow e^{i\phi} V_p$ and neutron $V_n \rightarrow e^{i\phi} V_n$ amplitudes to be transformed simultaneously. Indeed, in this case the density matrix and pairing tensor transform as:

$$\rho \equiv V^* V^T \longrightarrow \rho \quad \text{and} \quad \kappa \equiv V^* U^T \longrightarrow e^{-i\phi} \kappa. \quad (3)$$

Hence, the single-particle potential $\mathbf{\Gamma} \propto \boldsymbol{\rho}$ and pairing potential $\boldsymbol{\Delta} \propto \boldsymbol{\kappa}$ become

$$\mathbf{\Gamma} \longrightarrow \mathbf{\Gamma} \quad \text{and} \quad \boldsymbol{\Delta} \longrightarrow e^{-i\phi} \boldsymbol{\Delta}. \quad (4)$$

The GGI allows, for example, to choose one of the pairing gaps to be real and we will take advantage of it by assuming [apart of Sec. IV] that the neutron gap $\Delta_{nn} = \Delta > 0$. However, in some cases and for sake of simplicity, the eigenvectors will be given only up to the gauge transformation.

III. TWO-DIMENSIONAL ISO-CRANKING SOLUTIONS OF THE $t = 1$ PAIRING MODEL

Let us consider in this section a model hamiltonian [$\hbar \equiv 1$ for convenience]:

$$\hat{H}^{\omega_\tau} = \hat{h}_{sp} - G_{t=1} \hat{\mathbf{P}}_1^\dagger \hat{\mathbf{P}}_1 - \vec{\omega}_\tau \vec{t}, \quad (5)$$

containing an iso-symmetric particle-hole (ph) mean-field \hat{h}_{sp} . The isovector $\mathbf{t}=1$ pairing interaction is generated by:

$$\hat{\mathbf{P}}_{1\pm 1}^\dagger = \sum_{i>0} \hat{a}_{in(p)}^\dagger \hat{a}_{in(p)}^\dagger \quad \text{and} \quad \hat{\mathbf{P}}_{10}^\dagger = \frac{1}{\sqrt{2}} \sum_{i>0} (\hat{a}_{in}^\dagger \hat{a}_{ip}^\dagger + \hat{a}_{ip}^\dagger \hat{a}_{in}^\dagger), \quad (6)$$

and we consider two-dimensional iso-rotations $\vec{\omega}_\tau = [\omega_\tau \cos \varphi, \omega_\tau \sin \varphi, 0]$. Planar rotation in $N=Z$ nuclei are the most general since $\langle \hat{t}_z \rangle = 0$ due to number conservation. We are interested in analytical but non-selfconsistent BCS solutions within the constant gap approximation. Moreover, we assume that $\lambda_n = \lambda_p = \lambda$ and $|\Delta_{nn}| = \Delta = |\Delta_{pp}|$ but do not make any further restrictions concerning the phase relations between the gaps. Within the BCS approximation and under the above mentioned constraints the problem reduces to a diagonalisation of a 4×4 matrix with Δ and Δ_o being real [$\tilde{e}_i \equiv e_i - \lambda$]:

$$\begin{bmatrix} \tilde{e}_i - E_i & -\frac{1}{2}\omega_\tau e^{-i\varphi} & \Delta & e^{i\psi} \Delta_o \\ -\frac{1}{2}\omega_\tau e^{i\varphi} & \tilde{e}_i - E_i & e^{i\psi} \Delta_o & e^{i\alpha} \Delta \\ \hline \Delta & e^{-i\psi} \Delta_o & -\tilde{e}_i - E_i & \frac{1}{2}\omega_\tau e^{i\varphi} \\ e^{-i\psi} \Delta_o & e^{-i\alpha} \Delta & \frac{1}{2}\omega_\tau e^{-i\varphi} & -\tilde{e}_i - E_i \end{bmatrix} \begin{bmatrix} U_{i,n} \\ U_{i,p} \\ \hline V_{i,n} \\ V_{i,p} \end{bmatrix} = 0 \quad (7)$$

The physical (positive) eigenvalues of (7) are:

$$E_{i\pm} = \sqrt{\tilde{e}_i^2 + \frac{1}{4}\omega_\tau^2 + \Delta_o^2 + \Delta^2 \pm \sqrt{X_i}} \quad (8)$$

where

$$\begin{aligned} X_i = & \tilde{e}_i^2 \omega_\tau^2 + 4\Delta^2 \Delta_o^2 \cos^2 \left(\psi - \frac{\alpha}{2} \right) + \omega_\tau^2 \Delta^2 \sin^2 \left(\varphi - \frac{\alpha}{2} \right) \\ & - 4\tilde{e}_i \omega_\tau \Delta \Delta_o \cos \left(\varphi - \frac{\alpha}{2} \right) \cos \left(\psi - \frac{\alpha}{2} \right) \end{aligned} \quad (9)$$

These roots are double-degenerated [Kramers degeneracy] with eigenvectors of the form:

$$\begin{bmatrix} U_{i\pm,n} \\ U_{i\pm,p} \\ U_{i\pm,n}^\tau \\ U_{i\pm,p}^\tau \\ V_{i\pm,n} \\ V_{i\pm,p} \\ V_{i\pm,n}^\tau \\ V_{i\pm,p}^\tau \end{bmatrix} : \longrightarrow \begin{bmatrix} 0 \\ 0 \\ U_{i\pm,n}^\tau \\ U_{i\pm,p}^\tau \\ V_{i\pm,n} \\ V_{i\pm,p} \\ 0 \\ 0 \end{bmatrix} ; \quad \begin{bmatrix} U_{i\pm,n}^\tau \\ U_{i\pm,p}^\tau \\ 0 \\ 0 \\ 0 \\ 0 \\ -V_{i\pm,n} \\ -V_{i\pm,p} \end{bmatrix} . \quad (10)$$

A. Model including $t=1$, $t_z=\pm 1$ pairing

It is pedagogical to consider solutions to the model step by step starting with the standard case of a pn unpaired system $\Delta_o = 0$. In this case the eigenvalues reduce to:

$$E_{i\pm} = \sqrt{\Delta^2 \cos^2 \left(\varphi - \frac{\alpha}{2} \right) + \left[\frac{\omega_\tau}{2} \pm \sqrt{\tilde{e}_i^2 + \Delta^2 \sin^2 \left(\varphi - \frac{\alpha}{2} \right)} \right]^2} . \quad (11)$$

Searching for the eigenvectors we assume that: (i) Satisfactory solutions must obey a *minimal consistency condition* and reproduce the initial [see eq. (7)] relative phase relation for the gaps: $\Delta_{nn} = e^{i\alpha} \Delta_{pp}$. (ii) Moreover, we assume that neutron and proton amplitudes can differ only by their phase i.e.

$$U_{i,n} = e^{i\zeta} U_{i,p} \quad \text{and} \quad V_{i,n} = e^{i\eta} V_{i,p} \quad (12)$$

which appears to be a reasonable assumption especially for systems described by common state independent order parameter Δ . Under these conditions we encounter only two types

of solutions:

$$\varphi - \frac{\alpha}{2} = (2k+1)\frac{\pi}{2} \quad (13)$$

$$\varphi - \frac{\alpha}{2} = (2k)\frac{\pi}{2} \quad (14)$$

In the first case $\varphi - \alpha/2 = (2k+1)\pi/2$ the quasi-particle (qp) routhians (11) become linear as a function of iso-frequency:

$$E_{j\pm} = \left| E_j \pm \frac{1}{2}\omega_\tau \right| \quad \text{where} \quad E_j = \sqrt{\tilde{e}_j^2 + \Delta^2}. \quad (15)$$

Before the first band crossing i.e. for frequencies lower than

$$\omega_\tau \leq 2E_1 \equiv \omega_\tau^{(1)} \quad (16)$$

where E_1 denotes energy of the lowest qp at frequency zero, the qp routhians are

$$E_{j\pm} = E_j \pm \frac{1}{2}\omega_\tau. \quad (17)$$

The associated eigenvectors are also independent on ω_τ and equal:

$$\begin{bmatrix} U_{j\pm,n} \\ U_{j\pm,p} \\ V_{j\pm,n} \\ V_{j\pm,p} \end{bmatrix} : = \begin{bmatrix} U_j \\ \mp e^{i\varphi} U_j \\ V_j \\ \pm e^{-i\varphi} V_j \end{bmatrix} \quad (18)$$

where the V_j and U_j amplitudes are:

$$V_j = \frac{1}{2} \sqrt{1 - \frac{\tilde{e}_j}{E_j}} \quad \text{and} \quad U_j = \frac{1}{2} \frac{\Delta}{\sqrt{E_j(E_j - \tilde{e}_j)}}. \quad (19)$$

For frequencies $\omega_\tau^{(1)} \leq \omega_\tau \leq \omega_\tau^{(2)} \equiv 2E_2$ etc. standard procedure for blocked states can be applied to calculate eigenvectors, see e.g. [22].

The situation described here is characteristic for *non-collective* type of rotation. Indeed, in this case the iso-alignment will change stepwise at each crossing frequency $\omega_\tau^{(i)}$ similar to the sp-model described in detail in Refs. [14, 15, 16].

The second possibility (14) $\varphi - \alpha/2 = k\pi$ yields routhians of standard BCS-type:

$$E_{i\pm} = \sqrt{\tilde{e}_{j\pm}^2 + \Delta^2}. \quad (20)$$

where $\tilde{e}_{j\pm} \equiv \tilde{e}_j \pm \frac{1}{2}\omega_\tau$. The eigenvectors are:

$$\begin{bmatrix} U_{\bar{j}\pm,n} \\ U_{\bar{j}\pm,p} \\ V_{j\pm,n} \\ V_{j\pm,p} \end{bmatrix} : = \begin{bmatrix} U_{j\pm} \\ \mp e^{i\varphi} U_{j\pm} \\ V_{j\pm} \\ \mp e^{-i\varphi} V_{j\pm} \end{bmatrix} \quad (21)$$

where

$$V_{j\pm} = \frac{1}{2} \sqrt{1 - \frac{\tilde{e}_{j\pm}}{E_{j\pm}}} \quad \text{and} \quad U_{j\pm} = \frac{1}{2} \frac{\Delta}{\sqrt{E_{j\pm}(E_{j\pm} - \tilde{e}_{j\pm})}} \quad (22)$$

In this case the qp-routhians have a nontrivial dependence on ω_τ . They give rise to a smooth alignment processes typical for *collective* rotation. The particular case of this class of solutions corresponding to $\varphi = \alpha = 0$ was discussed in detail in ref. [16].

These two classes of solutions have a simple geometrical interpretation+cite[Gin68,Fra99]. Let us define the vector of anisotropy in isospace $\vec{\Delta} = [\Delta_x, \Delta_y, \Delta_z]$ as:

$$\Delta_x = \frac{1}{\sqrt{2}}(\Delta_{pp} - \Delta_{nn}); \quad \Delta_y = \frac{-i}{\sqrt{2}}(\Delta_{pp} + \Delta_{nn}); \quad \Delta_z = \Delta_{pn} \quad (23)$$

In our case $\Delta_o = 0$ and:

$$\frac{\Delta_x}{\Delta_y} = -\tan\left(\frac{\alpha}{2}\right). \quad (24)$$

as shown schematically in fig. 1. Relations (13)-(14) position the axis of iso-rotation either parallel or perpendicular with respect to $\vec{\Delta}$ giving rise to non-collective or collective iso-rotation, respectively. In particular, for most standard choices of phases $\Delta_{nn} = \pm\Delta_{pp}$ the collective axis is the $x(y)$ -axis, respectively. One may summarize that the possible solutions in this case only allow for principal axis cranking, where the cranking axis is determined by the gauge angle of the neutron and proton pair gap, respectively.

B. Model including complete $t=1$ pairing

According to the geometrical interpretation of the pairing gaps, switching on adiabatically pn pairing $\Delta_{pn}[\equiv \Delta_z]$ should always induce *collectivity*. Indeed, in this case the qp routhians (8) would depend on ω_τ in a complicated, nonlinear way for both cases (13) and (14). Closer examination shows, however, that the *non-collective* solution (18) cannot be generalized to accomodate pn pairing. [47] On the contrary, *collective* solutions (21) can be rather

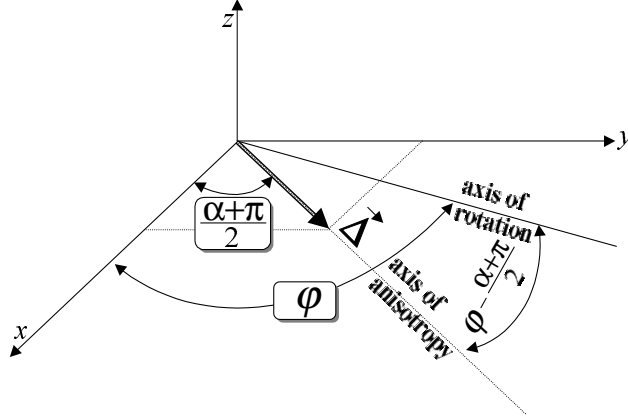


FIG. 1: Schematic drawing showing the relative position of the axis of anisotropy $[\vec{\Delta}]$ versus axis of isorotation. The phase relations (13)-(14) allow either for parallel [non-collective case] or perpendicular [collective case] position of the rotation axes only. Note that tilted solutions are not allowed and hence, three-dimensional isocranking in $N=Z$ nuclei can be effectively reduced to one-dimensional theory.

straightforwardly extended to include pn pairing. In this case the qp routhians take the form:

$$E_{j\pm} = \sqrt{\tilde{e}_{j\pm}^2 + |\Delta \mp \Delta_o e^{i(\psi-\varphi)}|^2}. \quad (25)$$

and the eigenvectors are:

$$\begin{bmatrix} U_{j\pm,n} \\ U_{j\pm,p} \\ V_{j\pm,n} \\ V_{j\pm,p} \end{bmatrix} : = \begin{bmatrix} U_{j\pm} \\ \mp e^{i\varphi} U_{j\pm} \\ V_{j\pm} \\ \mp e^{-i\varphi} V_{j\pm} \end{bmatrix} \quad (26)$$

where

$$V_{j\pm} = \frac{1}{2} \sqrt{1 - \frac{\tilde{e}_{j\pm}}{E_{j\pm}}} \quad \text{and} \quad U_{j\pm} = \frac{1}{2} \frac{\Delta \mp \Delta_{pn} e^{-i\varphi}}{\sqrt{E_{j\pm}(E_{j\pm} - \tilde{e}_{j\pm})}}. \quad (27)$$

Let us observe that for $\psi - \varphi = (2n + 1)\pi/2$ the routhians (25) have a similar form like discussed by Goodman [23] for the static case of $\omega_\tau=0$:

$$E_{i\pm} = \sqrt{\tilde{e}_{i\pm}^2 + \Delta^2 + \Delta_o^2}. \quad (28)$$

For $\psi - \varphi = n\pi$, on the other hand, one gets routhians similar in structure to those discussed by Bes *et al.* [20]:

$$E_{i\pm} = \sqrt{\tilde{e}_{i\pm}^2 + (\Delta \mp (-1)^n \Delta_o)^2}. \quad (29)$$

To gain further inside into our solutions let us calculate the density matrix:

$$\rho_{i\tau, i\tau} = \rho_{\bar{i}\tau, \bar{i}\tau} = \frac{1}{4} \left\{ 2 - \frac{\tilde{e}_{i+}}{E_{i+}} - \frac{\tilde{e}_{i-}}{E_{i-}} \right\} \quad \text{for } \tau = n, p \quad (30)$$

$$\rho_{in, ip} = \rho_{\bar{i}n, \bar{i}p} = \frac{e^{-i\varphi}}{4} \left\{ \frac{\tilde{e}_{i+}}{E_{i+}} - \frac{\tilde{e}_{i-}}{E_{i-}} \right\} \quad (31)$$

and pairing tensor:

$$\kappa_{jn, \bar{j}n} = e^{-i\alpha} \kappa_{jp, \bar{j}p} = \frac{1}{4} \Delta X_j^{(+)} - \frac{1}{4} \Delta_o e^{i(\psi-\varphi)} X_j^{(-)} \quad (32)$$

$$\kappa_{in, \bar{i}p} = \kappa_{ip, \bar{i}n} = \frac{1}{4} \Delta_o e^{i\psi} X_j^{(+)} - \frac{1}{4} \Delta e^{i\varphi} X_j^{(-)} \quad (33)$$

where

$$X_i^{(\pm)} = \frac{1}{E_{i+}} \pm \frac{1}{E_{i-}} \quad \text{and} \quad X^{(\pm)} = \sum_i X_i^{(\pm)} \quad (34)$$

Then, the gap equations are:

$$\frac{4}{G_{t=1, t_z=\pm 1}} = X^{(+)} - \frac{\Delta_o}{\Delta} e^{i(\psi-\varphi)} X^{(-)} \quad (35)$$

$$\frac{4}{G_{t=1, t_z=0}} = X^{(+)} - \frac{\Delta}{\Delta_o} e^{-i(\psi-\varphi)} X^{(-)}. \quad (36)$$

For isospin symmetric pairing $G_{t=1, t_z=\pm 1} = G_{t=1, t_z=0} = G_{t=1}$ the equations can be solved either when: **(a)** $X^{(-)} = 0$ and for essentially arbitrary values of gaps or **(b)** for $\Delta = \Delta_o$ and $\psi - \varphi = n\pi$. The latter case leads to gapless superconductivity (29) with a rather unphysical qp-spectrum.

In particular, for the equidistant $e_i = i\delta\epsilon$ level model with symmetric cut-off, the particle-hole symmetry leads to $X^{(-)} = 0$ but only for the routhians of Goodman-type (28). In such a case the particle number condition $N = \text{Tr}\rho$ is also automatically satisfied since [see eq. (30)]:

$$\sum_i \left\{ \frac{\tilde{e}_{i+}}{E_{i+}} + \frac{\tilde{e}_{i-}}{E_{i-}} \right\} \equiv 0. \quad (37)$$

The alignment in iso-space is then given by the formula:

$$T = \sqrt{\langle \hat{t}_x \rangle^2 + \langle \hat{t}_y \rangle^2} = \frac{1}{2} \sum_{i>0} \left\{ \frac{\tilde{e}_{i+}}{E_{i+}} - \frac{\tilde{e}_{i-}}{E_{i-}} \right\} \quad (38)$$

which is analogous to the one obtained in ref. [16] [there, Δ^2 should be formally replaced by $\Delta^2 + \Delta_o^2$]. Moreover, since for the isospin symmetric model $\Delta^2 + \Delta_o^2$ is constant the moment of inertia in isospace (MoI-i) and all conclusions drawn there remain unaffected.

For more complex routhians (e.g. given by eq. (29) we have not been able find analytical solutions even for such high-symmetry like the equidistant level model.

IV. MODELS INCLUDING ISOSCALAR PAIRING.

A. Pure $t=0$ pairing model.

Let us now consider the model hamiltonian:

$$\hat{H}^{\omega_\tau} = \hat{h}_{sp} - G_{t=1} \hat{\mathbf{P}}_1^\dagger \hat{\mathbf{P}}_1 - G_{t=0} \hat{\mathbf{P}}_0^\dagger \hat{\mathbf{P}}_0 - \vec{\omega}_\tau \vec{\mathbf{t}}, \quad (39)$$

consisting of both isovector and isoscalar pairing interactions coupling particles in time reversed orbits:

$$\mathbf{P}_0^\dagger = \frac{1}{\sqrt{2}} \sum_{i>0} (a_{in}^\dagger a_{ip}^\dagger - a_{ip}^\dagger a_{in}^\dagger). \quad (40)$$

Let us start our considerations with a pure isoscalar $t = 0$ model. In this case the BCS equations take the following form [the pairing gap Δ_0 can be chosen real]:

$$\left[\begin{array}{cc|cc} \tilde{e}_i - E_i & -\frac{1}{2}\omega_\tau e^{-i\varphi} & & -\Delta_0 \\ -\frac{1}{2}\omega_\tau e^{i\varphi} & \tilde{e}_i - E_i & & \Delta_0 \\ \hline & & \Delta_0^* & \\ -\Delta_0^* & & & \end{array} \right] \begin{bmatrix} U_{i,n} \\ U_{i,p} \\ V_{i,n} \\ V_{i,p} \end{bmatrix} = 0 \quad (41)$$

The eigenvalues are linear in ω_τ and equal:

$$E_{i\pm} = \left| \frac{1}{2}\omega_\tau \pm E_i \right| = E_i \pm \frac{1}{2}\omega_\tau \quad \text{where} \quad E_i = \sqrt{\tilde{e}_i^2 + \Delta_0^2}. \quad (42)$$

The equations are valid below the first crossing frequency $\omega_\tau \leq \omega_\tau^{(1)} \equiv 2E_1$. The roots are double-degenerated due to the Kramers degeneracy and equal:

$$\begin{bmatrix} U_{i\pm,n} \\ U_{i\pm,p} \\ U_{\bar{i}\pm,n} \\ U_{\bar{i}\pm,p} \\ V_{i\pm,n} \\ V_{i\pm,p} \\ V_{\bar{i}\pm,n} \\ V_{\bar{i}\pm,p} \end{bmatrix} \longrightarrow \begin{bmatrix} 0 \\ 0 \\ U_i \\ \mp e^{i\varphi} U_i \\ V_i \\ \pm e^{-i\varphi} V_i \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} U_i \\ \mp e^{i\varphi} U_i \\ 0 \\ 0 \\ 0 \\ 0 \\ V_i \\ \pm e^{-i\varphi} V_i \end{bmatrix} \quad (43)$$

where

$$U_i = \frac{1}{2} \sqrt{1 + \frac{\tilde{e}_i}{E_i}} \quad \text{and} \quad V_i = \frac{1}{2} \sqrt{1 - \frac{\tilde{e}_i}{E_i}} \quad (44)$$

independently on ω_τ . The situation is similar to that described by (15)-(18) for the case of $\mathbf{t}=1$, $\mathbf{t}_z=\pm 1$ pairing in Sec. III A. However, since the pair field is isotropic in isospace only non-collective iso-rotation takes place regardless of the direction of the iso-cranking axis.

B. The $\mathbf{t}=0$ plus $\mathbf{t}=1$, $\mathbf{t}_z=\pm 1$ pairing model.

The extension of the pure $\mathbf{t}=0$ model including $\mathbf{t}=1$, $\mathbf{t}_z=\pm 1$ can be done by either linking it smoothly to *non-collective* solutions (18) or *collective* solutions (21). Let us consider first *non-collective* solutions obeying the condition (13). These can be relatively easy generalized to include $\mathbf{t}=0$ pairing provided that the following phase relation is satisfied:

$$\theta - \frac{1}{2}\alpha = n\pi \quad (45)$$

where $\Delta_0 = e^{i\theta} |\Delta_0|$. This is a necessary condition for the linear term of the determinant of the BCS matrix to vanish and, in turn, to preserve the mirror symmetric structure $[E_i, -E_i]$ of the HFB solutions. In this case our solutions have *non-collective* character. The quasi-particle routhians below the first crossing frequency take the following form:

$$E_{i\pm} = E_i \pm \frac{1}{2}\omega_\tau \quad \text{where} \quad E_i = \sqrt{\tilde{e}_i^2 + \Delta^2 + |\Delta_0|^2} \quad (46)$$

The associated eigenvectors are:

$$\begin{bmatrix} U_{i\pm,n} \\ U_{i\pm,p} \\ U_{i\pm,n} \\ U_{i\pm,p} \\ V_{i\pm,n} \\ V_{i\pm,p} \\ V_{i\pm,n} \\ V_{i\pm,p} \end{bmatrix} \longrightarrow \begin{bmatrix} 0 \\ 0 \\ U_{i\pm} \\ \mp e^{i\varphi} U_{i\pm} \\ V_{i\pm} \\ \pm e^{-i\varphi} V_{i\pm} \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} -U_{i\pm}^* \\ \pm e^{i\varphi} U_{i\pm}^* \\ 0 \\ 0 \\ 0 \\ 0 \\ V_{i\pm} \\ \pm e^{-i\varphi} V_{i\pm} \end{bmatrix}. \quad (47)$$

where

$$V_{i\pm} = \frac{1}{2} \sqrt{1 - \frac{\tilde{e}_i}{E_i}}; \quad \text{and} \quad U_{i\pm} = \frac{1}{2} \frac{\Delta \mp e^{-i\varphi} \Delta_0}{\sqrt{E_i(E_i - \tilde{e}_i)}}. \quad (48)$$

However, since the pairing tensors are:

$$\kappa_{in,\tilde{in}} = e^{-i\alpha} \kappa_{ip,\tilde{ip}} = \frac{\Delta}{2E_i} \quad \text{and} \quad \kappa_{in,\tilde{ip}} = -\kappa_{ip,\tilde{in}} = \frac{\Delta_0}{2E_i} \quad (49)$$

the selfconsistency conditions for pairing gaps yield the additional constraint:

$$\frac{2}{G_{t=1}} = \sum_{i>0} \frac{1}{E_i} = \frac{2}{G_{t=0}} \quad (50)$$

for the coupling constants. Solutions are therefore possible only for $G_{t=1}=G_{t=0}$. Coexistence of this type was already reported in the literature [1, 2, 3].

Generalization of the *collective* solution (13) to include $\mathbf{t}=0$ pairing is far more difficult. It can be shown, however, that for the following phase relations:

$$\theta - \varphi = (2n + 1) \frac{\pi}{2}, \quad (51)$$

the eigenvectors take the following form:

$$\begin{bmatrix} U_{i\pm,n} \\ U_{i\pm,p} \\ U_{i\pm,n} \\ U_{i\pm,p} \\ V_{i\pm,n} \\ V_{i\pm,p} \\ V_{i\pm,n} \\ V_{i\pm,p} \end{bmatrix} \longrightarrow \begin{bmatrix} 0 \\ 0 \\ U_{i\pm} \\ \mp e^{i\varphi} U_{i\pm}^* \\ V_{i\pm} \\ \mp e^{-i\varphi} V_{i\pm}^* \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} -U_{i\pm}^* \\ \pm e^{i\varphi} U_{i\pm}^* \\ 0 \\ 0 \\ 0 \\ 0 \\ V_{i\pm}^* \\ \pm e^{-i\varphi} V_{i\pm}^* \end{bmatrix}. \quad (52)$$

In this case the quasiparticle routhians are:

$$E_{i\pm} = \left(\tilde{e}_i^2 + \frac{\omega_\tau^2}{4} + \Delta^2 + |\Delta_0|^2 \pm 2\sqrt{\frac{\omega_\tau^2}{4}(\tilde{e}_i^2 + |\Delta_0|^2) + \Delta^2|\Delta_0|^2} \right)^{1/2} \quad (53)$$

For the general case of $\omega_\tau \neq 0$ the amplitudes are given by lengthy and rather non-transparent expressions. However, the property of these solutions are easily recognized already at $\omega_\tau = 0$ when:

$$V_{i\pm} = \frac{e^{-i\frac{(\varphi-\theta)}{2}}}{\sqrt{2}} \sqrt{1 - \frac{\tilde{e}_i}{E_{i\pm}}} \cos \frac{\varphi - \theta}{2} \quad (54)$$

$$U_{i\pm} = \frac{e^{-i\frac{(\varphi-\theta)}{2}}}{\sqrt{2}} \frac{\Delta \pm |\Delta_0|}{\sqrt{E_{i\pm}(E_{i\pm} - \tilde{e}_i)}} \cos \frac{\varphi - \theta}{2} \quad (55)$$

and

$$E_{i\pm} = \sqrt{\tilde{e}_i^2 + (\Delta \pm |\Delta_0|)^2}. \quad (56)$$

The gap equations take the following form:

$$\frac{4}{G_{t=1}} = X^{(+)} + \frac{|\Delta_0|}{\Delta} X^{(-)} \quad (57)$$

$$\frac{4}{G_{t=0}} = X^{(+)} + \frac{\Delta}{|\Delta_0|} X^{(-)} \quad (58)$$

where $X^{(\pm)}$ are defined as in eq. (34). Since $X^{(-)} < 0$ the gap equations can be solved only when $G_{t=0}=G_{t=1}$ and $\Delta = |\Delta_0|$. In this case the quasiparticle spectrum takes the rather unphysical gapless form in spite of the strength of pairing force. Let us mention here that particle-number is in this case automatically satisfied at least for equidistant spectrum since in this case particle-hole symmetry [eq. (37)] is fulfilled.

V. ISOSPIN SYMMETRY BREAKING

The considerations of Sec. III show that *collective* iso-rotations are possible when the isovector pairing field $\vec{\Delta}$ is perpendicular to the axis of iso-cranking. Such a collective motion allows to take into account isospin fluctuations in a static way by replacing the cranking constraint:

$$\sqrt{\langle \hat{t}_x \rangle^2 + \langle \hat{t}_y \rangle^2} = T \quad (59)$$

by

$$\sqrt{\langle \hat{t}_x \rangle^2 + \langle \hat{t}_y \rangle^2} = \sqrt{T(T+1)} \quad (60)$$

In this way the Wigner energy [$E_W \sim T$] can be, at least partially, restored. Quantitative calculations [14, 15, 16] show, however, that in the presence of the standard $\mathbf{t}=1$ field, the MoI is too large and cannot account for the empirical data. In these calculations the isovector part of the *particle-hole* field is not taken into account [24] (see also Sec. VIII). In the absence of these correlations the mechanism which lowers the MoI in isospace was fully ascribed to isoscalar pairing in complete analogy to the mechanism of lowering the spatial MoI by the isovector superfluidity.

However, as demonstrated in Sect. IV, at the level of the BCS approximation one cannot obtain a mixed *collective* solution which in turn is necessary to lower the MoI and to account for isospin fluctuations in a static way (60). To make use of the static condition (60) and to restore (locally) the MoI in isospace it is necessary to find phase mixed solutions. It has already been shown [2] that approximate number projection of Lipkin-Nogami type allows for the mixing of $\mathbf{t}=1$ and $\mathbf{t}=0$ pairing phases, breaking the isospin symmetry. The mixing appears only when the strength $G_{t=0}$ of the average isoscalar field exceeds the strength of the isovector pair field $G_{t=1}$ i.e. for $x^{t=0} = G_{t=0}/G_{t=1} \geq x_{crit} \sim 1.1$ [48] The Wigner energy generated through this mechanism contributes with opposite sign to $\lambda_{\tau\tau}^{(2)}$ and $\lambda_{\tau-\tau}^{(2)}$ building up an asymmetry in the auxilliary LN fields:

$$\lambda_{\tau\tau'}^{(2)} \Delta N_{\tau} \Delta N_{\tau'}. \quad (61)$$

Since the LN solution does not allow for an isovector *pn*-field, the situation qualitatively resembles the case considered in Sec. IV B. Let us however point out that the LN procedure introduces an explicit state dependence of the effective gaps

$$\Delta_{i\tau\bar{j}\tau'} = \Delta_{\tau\tau'} \delta_{ij} + 2\lambda_{\tau\tau}^{(2)} \kappa_{i\tau\bar{j}\tau'}. \quad (62)$$

Moreover, the Lipkin Nogami parameters $\lambda_{\tau\tau'}^{(2)}$ are anisotropic in isospace. In turn, a wider class of solutions become possible, including solutions which mix $\mathbf{t}=1$ $\mathbf{t}=0$ pairing phases and are *collective* in isospace. The absence of the isovector *pn*-field in the LN solutions limits, however, the possibilities to calculate the amplitude of the isovector *pn*-pair transfer, see next section.

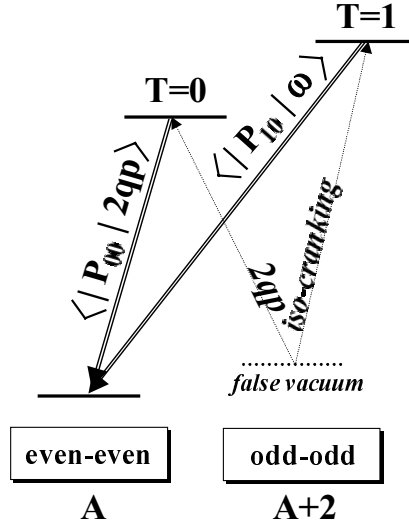


FIG. 2: Schematic figure of the pair transfer and structure of the $T=0$ and $T=1$ states in o-o ($A+2$) and ground state of e-e (A) $N=Z$ nuclei. The thin arrows indicate the structure of the $T=0(1)$ states in o-o nuclei in our model. Note, that the different structures of the $T=0$ states imply the quenching of isoscalar pair transfer even for a $t=0$ paired systems.

VI. PROTON-NEUTRON PAIR TRANSFER

The theory of pair transfer/stripping processes like $(\alpha, d), (^3\text{He}, n)$ etc. were developed already in the end of 50's and the beginning of 60's [25, 26, 27, 28] based on the plane wave Born approximation. The general expression for two-nucleon spectroscopic factor (2nSF) carrying nuclear structure information was derived using the spherical shell model in the jj -coupling limit, see Refs. [27, 28]. The first analysis of the 2nSF using the pairing interaction model was done by Yoshida [29]. He pointed out the possibility of an enhanced cross-section for the two-particle transfer of the isovector pair due to collective pairing phenomena which may be particularly strong if the pairing coherency extends over few j -shells. Based on Yoshida's work, Fröbrich [30, 31] has analysed the influence of pn pairing on pn -pair transfer in $N=Z$ nuclei using both $t=1$ and $t=0$ pairing interactions within a single- j shell model space. He pointed out that pn -pairing can enhance the cross-section by a factor of 3 as compared to conventional shell-model calculations of [32].

In the deformed shell model [paired mean-field] the differential cross-section describing

the transfer of a structureless isoscalar or isovector pn pair is proportional to:

$$\frac{d\sigma}{d\Omega} \sim |T_{10(00)}^{(A,A+2)}|^2 = |\langle \Psi_A | \mathbf{P}_{10(00)} | \Psi_{A+2}(T) \rangle|^2 \quad (63)$$

where we have assumed that the transfer goes from the $T = 1(0)$ ground state $|\Psi_{A+2}(T)\rangle$ of the o-o $N=Z=(A+2)/2$ nucleus to the ground state $|\Psi_A\rangle$ of the even-even nucleus $N=Z=A/2$. Within our model [14, 15, 16] $|\Psi_A\rangle$ has the structure of a 0qp state while the structure of $|\Psi_{A+2}(T)\rangle$ depends on the isospin T . For $T=0$, $|\Psi_{A+2}(T=0)\rangle$ is a two-quasiparticle state while the $|\Psi_{A+2}(T=1)\rangle$ states maintain the 0qp structure but is cranked in isospace, see fig. 2. In all cases the $|\Psi_A\rangle$ and $|\Psi_{A+2}(T)\rangle$ states are described by the fully self-consistent amplitudes $(\mathbf{U}^{(A)} \mathbf{V}^{(A)})$ and $(\mathbf{U}^{(A+2)} \mathbf{V}^{(A+2)})$. Therefore, one can make use of the generalized Wick's theorem to derive the explicit expression for the pair transfer amplitude $T^{(A,A+2)}$ (63):

$$T_{10(00)}^{(A,A+2)} = \frac{1}{\sqrt{2}} \langle \Psi_A | \Psi_{A+2} \rangle \sum_{i>0} \left\{ \kappa_{in,\bar{i}p}^{(A,A+2)} \pm \kappa_{ip,\bar{i}n}^{(A,A+2)} \right\} \quad (64)$$

where

$$\boldsymbol{\kappa}^{(A,A+2)} = (\mathbf{V}^{(A+2)})^* (\mathbf{U}^T)^{-1} (\mathbf{U}^{(A)})^T \quad (65)$$

$$\mathbf{U} \equiv (\mathbf{U}^{(A+2)})^\dagger \mathbf{U}^{(A)} + (\mathbf{V}^{(A+2)})^\dagger \mathbf{V}^{(A)}. \quad (66)$$

The overlap is given by Onishi formula:

$$\langle \Psi_A | \Psi_{A+2} \rangle = \sqrt{\text{Det} \mathbf{U}}. \quad (67)$$

According to our model there is a fundamental difference between the structure of the $T=0$ and $T=1$ states in the parent [o-o] nucleus, see fig. 2. The $T=0$ states correspond to 2QP excitations. Therefore the deuteron will be transferred from a 2QP state to the BCS vacuum in the daughter nucleus. In such a case the $T_{00}^{(A,A+2)}$ amplitude will essentially probe $\sim (U_A)_i^2$ [i enumerates blocked qp state] and will always be severely quenched [29].

On the other hand the isovector pn -pair will be transferred between 0qp states and can, in principle, be enhanced due to the pairing collectivity. The generic situation is illustrated in fig. 3 for the case of ^{48}Cr and ^{50}Mn . In these calculations we assume $x^{t=0} = 1.3$ for both the ground state of ^{48}Cr and the *false vacuum* of ^{50}Mn . With increasing ω_τ ^{50}Mn picks up a small fraction of the isovector pn pairing and $T_{10}^{(A,A+2)}$ increases reaching maximum around $T_x \sim 1$. Then in the region of the phase transition it decreases rapidly due to the vanishing

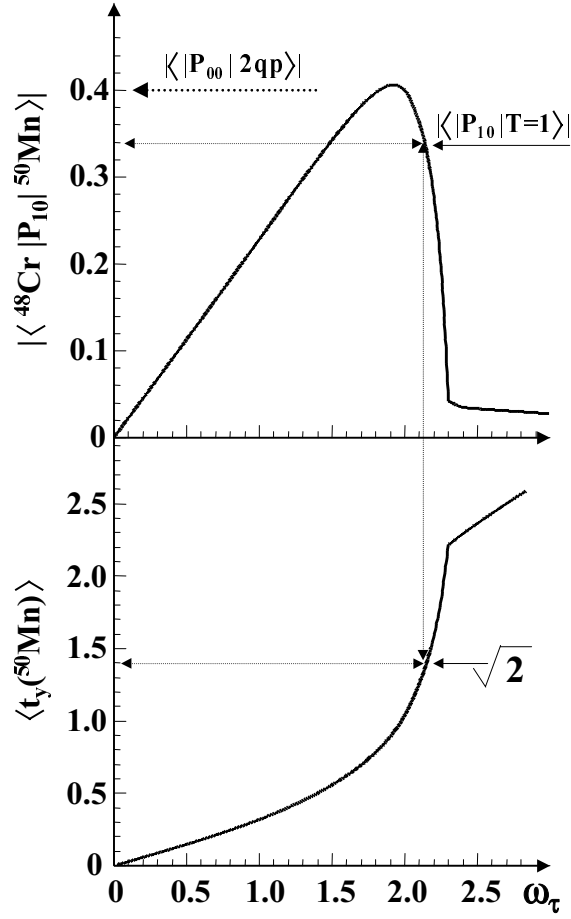


FIG. 3: Isovector pn -pair transfer $\langle P_{10} | \omega_\tau \rangle$ [upper part] and isoalignment [lower part] versus ω_τ . The calculations were done in the LN approximation. In this approximation $t=1$ pn pairing plays a redundant role [2, 13] resulting in the quenching of $\langle P_{10} | \omega_\tau \rangle$.

overlap, see eq. (64). In the region of $T_x \sim 1$ to $\sim \sqrt{2}$ the $|T_{10}^{(A,A+2)}| \sim |T_{00}^{(A,A+2)}| \sim 0.4$ i.e. no enhancement is calculated for the pair transfer.

The quenching of the $T_{10}^{(A,A+2)}$ amplitude is related to the fact that within the LN model the isovector pn pairing plays essentially a redundant role. To investigate the importance of isovector pn correlations we have to come back to the $t=1$ pairing model of Sec. III B. In this case an analytical expression for the transfer amplitude can be easily derived:

$$\frac{T_{10}^{(A,A+2)}}{\langle \Psi_A | \Psi_{A+2} \rangle} = \frac{e^{i\varphi}}{\sqrt{2}} \sum_{i>0} \left\{ \frac{u_{i+}^{(A)*} v_{i+}^{(A+2)}}{X_i^{(-)}} - \frac{u_{i-}^{(A)*} v_{i-}^{(A+2)}}{X_i^{(+)}} \right\} \quad (68)$$

where

$$X_i^{(\pm)} = u_{i\pm}^{(A)*} u_{i\pm}^{(A+2)} + v_{i\pm}^{(A)*} v_{i\pm}^{(A+2)}. \quad (69)$$

The amplitudes (\mathbf{u}, \mathbf{v}) define the Bogolyubov transformation in the canonical basis:

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|n\rangle \pm e^{i\varphi}|p\rangle) \quad (70)$$

$$|\widetilde{\pm}\rangle = \frac{1}{\sqrt{2}}(|\overline{n}\rangle \pm e^{i\varphi}|\overline{p}\rangle). \quad (71)$$

They are equal:

$$u_{i\pm} = \sqrt{2}U_{i\pm}^* \quad \text{and} \quad v_{i\pm} = \sqrt{2}V_{i\pm}^* \quad (72)$$

where $U_{i\pm}, V_{i\pm}$ are given by eq. (27) [for the daughter nucleus they are obtained at $\omega_\tau=0$]. Let us now assume that $\omega_\tau=0$. When $\Delta_{pn}=0$ then both $u_+ = u_-$ and $v_+ = v_-$ and we have an exact cancellation of two relatively large terms in the r.h.s of eq. (68). On the other hand, for $\Delta_{pn} \neq 0$, also $u_+ \neq u_-$ and we have a positive interference originating entirely from the terms proportional to Δ_{pn} and, in turn, a rapid increase of $|T^{(A,A+2)}|$ as a function of the contribution of the pn pair gap to the total gap $\Delta_T^2 \equiv \Delta^2 + |\Delta_{pn}|^2 = \text{const.}$ This is shown in fig. 4 in version a) In this version of the calculations $\alpha_{pn}^2 [|\Delta_{pn}|^2 \equiv \alpha_{pn}^2 \Delta_T^2]$ was forced to be the same for ^{48}Cr and ^{50}Mn (at $\omega_\tau = 0$). Version b), on the other hand, assumes fixed structure $\Delta = |\Delta_{pn}|$ in ^{48}Cr . In this case α_{pn}^2 refers to ^{50}Mn . It is clearly seen from the figure that the maximum transfer is expected for similar [but no necessarily equal] content of pn pairing in the parent and daughter nucleus. Obviously, other factors like different deformations etc. may additionally hinder pn -transfer.

VII. TOTAL ROUTHIAN SURFACE CALCULATIONS INCLUDING ISOSCALAR PAIRING

We have performed systematic calculations of the energy differences $\Delta E_T \equiv E_{T=0} - E_{T=1}$ between the lowest $T = 0$ and $T = 1$ states in o-o $N=Z$ nuclei using the Total Routhian Surface (TRS) method. We used the deformed Woods-Saxon potential and included both isovector and isoscalar pairing treated in the LN approximation. The deformation space covered quadrupole β_2 , γ and hexadecapole β_4 shapes. The liquid-drop formula of [33] was used to calculate the macroscopic part of the total energy. Apart of including $\mathbf{t}=0$ pairing, the details of the method follow our standard implementation and we refer the reader to Ref. [34] for further details.

The calculations have been done for all o-o $N=Z$ nuclei from ^{18}F to ^{74}Rb , see table I. To account for the $T = 0$ states the 2QP surfaces $\alpha_1^\dagger \alpha_2^\dagger |0\rangle$ have been created where $\alpha_{1(2)}^\dagger$

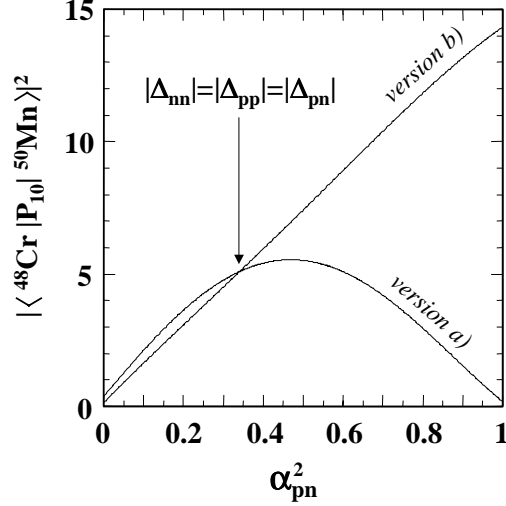


FIG. 4: Pair transfer amplitude $\langle |P_{10}| \rangle$ calculated in the BCS approximation for a model involving $t=1$ correlations only. The results are shown as a function of the contribution of Δ_{pn} to the total gap, see text for details.

denote the lowest qp excitations of the same signature. We use the notion of signature rather than time-reversal since the calculations have been done at a small fixed spatial rotational frequency. By blocking $\alpha_1^\dagger \alpha_2^\dagger |0\rangle$ we assure that all pairing fields are truly blocked. Indeed, blocking the lowest qp states of different signatures $\alpha_1^\dagger \alpha_{\bar{1}}^\dagger |0\rangle$ results in the blocking of isovector pp and nn pairing but not isoscalar pn pairing. It is assumed that the valence blocked quasiparticles [proton-neutron] may interact. This interaction is added as a perturbation assuming a simple [isoscalar] delta force $g(A)_{eff} \delta(\mathbf{r}_1 - \mathbf{r}_2)$ as a residual interaction. For further simplification we assume either spherical $|(nljm)^2; I\rangle$ or Nilsson asymptotic $|(Nn_z \Lambda K)^2\rangle$ limits for the two-body wave function with $I = I_{exp}$ or $I_{exp} = 2K$, respectively. An effective strength of the residual force $g(A)_{eff} = g_{eff}/\sqrt{A}$ was assumed. This gives rise to a $\sim 1/A$ dependence for the matrix element in accordance to the standard mass dependence of the valence pn interaction in liquid-drop models [40].

The $T = 1$ states were calculated in a slightly simplified manner. Self-consistent TRS calculations have been done only for the 0QP state [*false vacuum*] at $\omega_\tau=0$. Then, the isocranking calculations were performed at the deformation minimum of the TRS calculations.

The TRS calculations have been performed for a slightly reduced strength of the pairing

	$E_1^{T=0}$	$E_2^{T=0}$	$E_1^{T=1}$	$E_2^{T=1}$
${}^6_3\text{Li}_3$	0 (1 ⁺)	2.185 (3 ⁺)	3.562 (0 ⁺)	5.370 (2 ⁺)
${}^{10}_5\text{B}_5$	0 (3 ⁺)	0.718 (1 ⁺)	1.742 (0 ⁺)	5.163 (2 ⁺)
${}^{14}_7\text{N}_7$	0 (1 ⁺)	4.915 (0 ⁻)	2.313 (0 ⁺)	8.062 (1 ⁻)
${}^{18}_9\text{F}_9$	0 (1 ⁺)	0.937 (3 ⁺)	1.041 (0 ⁺)	3.061 (2 ⁺)
${}^{22}_{11}\text{Na}_{11}$	0 (3 ⁺)	0.583 (1 ⁺)	0.657 (0 ⁺)	1.952 (2 ⁺)
${}^{26}_{13}\text{Al}_{13}$	0 (5 ⁺)	0.417 (3 ⁺)	0.228 (0 ⁺)	2.070 (2 ⁺)
${}^{30}_{15}\text{P}_{15}$	0 (1 ⁺)	0.709 (1 ⁺)	0.677 (0 ⁺)	2.938 (2 ⁺)
${}^{34}_{17}\text{Cl}_{17}$	0.146 (3 ⁺)	0.461 (1 ⁺)	0 (0 ⁺)	2.158 (2 ⁺)
${}^{38}_{19}\text{K}_{19}$	0 (3 ⁺)	0.459 (1 ⁺)	0.130 (0 ⁺)	2.403 (2 ⁺)
${}^{42}_{21}\text{Sc}_{21}$	0.611 (1 ⁺)	0.617 (7 ⁺)	0 (0 ⁺)	1.586 (2 ⁺)
${}^{46}_{23}\text{V}_{23}$	0.801 (3 ⁺)	0.915 (1 ⁺)	0 (0 ⁺)	0.993 (2 ⁺)
${}^{50}_{25}\text{Mn}_{25}$	0.230 (5 ⁺)	0.651 (1 ⁺)	0 (0 ⁺)	0.800 (2 ⁺)
${}^{54}_{27}\text{Co}_{27}$	0.199 (7 ⁺)	0.937 (1 ⁺)	0 (0 ⁺)	1.447 (2 ⁺)
${}^{58}_{29}\text{Cu}_{29}$	0 (1 ⁺)	0.445 (1 ⁺)	0.203 (0 ⁺)	1.652 (2 ⁺)
${}^{62}_{31}\text{Ga}_{31}$	0.571 (1 ⁺)	0.818 (3 ⁺)	0 (0 ⁺)	
${}^{66}_{33}\text{As}_{33}$	0.837 [1 ⁺]	1.231 (3 ⁺)	0 (0 ⁺)	0.963 [2 ⁺]
${}^{70}_{35}\text{Br}_{35}$	1.337 (3 ⁺)	1.653 (5 ⁺)	0 (0 ⁺)	0.934 (2 ⁺)
${}^{74}_{37}\text{Rb}_{37}$	1.006 [3 ⁺]	1.224 [4 ⁺]	0 [0 ⁺]	0.478 [2 ⁺]

TABLE I: The lowest $T = 0$ and $T = 1$ states in odd-odd $N=Z$ nuclei. Data are taken from: ${}^{18}\text{F}$ – ${}^{58}\text{Cu}$ Ref. [35]; ${}^{62}\text{Ga}$ Ref. [36]; ${}^{66}\text{As}$ Ref. [37]; ${}^{70}\text{Br}$ Ref. [38]; ${}^{74}\text{Kr}$ Ref. [39]. Spin values in square parantheses are uncertain.

force as compared to the estimate of [14]. The TRS and δ -force corrected TRS results are presented in fig. 5. The strength of the δ -force $g_{eff} \sim 650 \text{ MeV}$ was estimated from a least square fit to the data. The size of the matrix element is more or less consistent with average liquid-drop estimate of $\sim 20/A \text{ MeV}$ of the pn effect for valence particles in o-o nuclei although one would expect an enhancement in $N=Z$ nuclei purely due to geometrical reason (congruence effect). However, the overall quality of the fit does not change very much even if we double g_{eff} and therefore is not very conclusive.

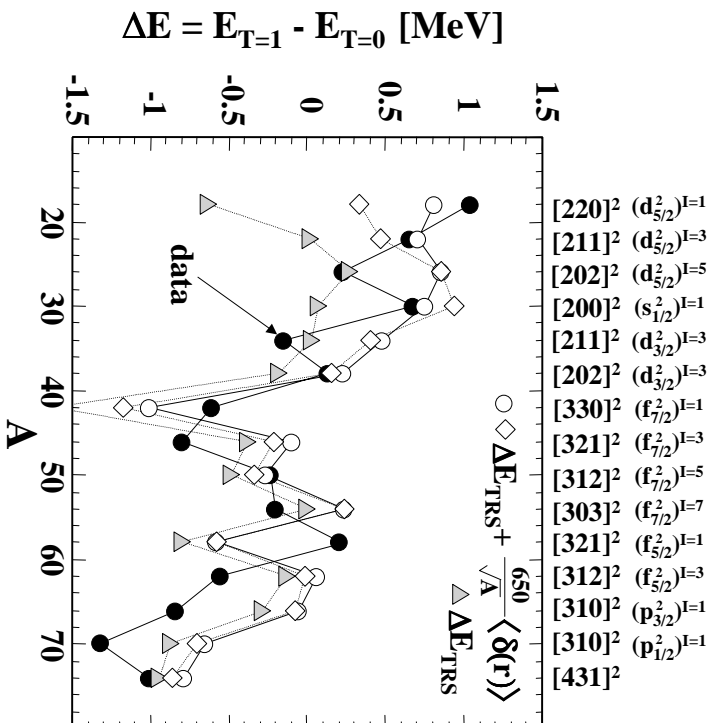


FIG. 5: The difference $\Delta E_T \equiv E_{T=1} - E_{T=0}$ calculated using the TRS model [triangles]. Open circles and diamonds mark the calculations including the correction due to the interaction of the valence particles evaluated assuming Nilsson asymptotic and spherical wave functions, respectively [see text]. Solid circles mark the available data which are collected in Table I.

The new elements in these calculations with respect to the one presented in Ref. [15] can be summarized as follows: (i) the equilibrium deformation is calculated from the TRS minimum (ii) two quasiparticles of the same type (i.e. not in time reversed states) have been blocked and the residual pn interaction between the valence pair has been added to the energy difference to reduce the coherence of the $t=0$ pair field. These new calculations describe well the global decrease of $\Delta E(A)$ including the sign inversion in the $f_{7/2}$ shell, see Fig 5. However, the details are far from being satisfactory reproduced. More extended studies of the wave function of the valence nucleons is not expected to improve the situation since the TRS ground states are essentially spherical for light nuclei. Deformation sets in only for heavy nuclei where spherical and asymptotic matrix elements of the δ -force are anyhow very similar. This seems to indicate that the $t=0$ pairing is too strong and needs further reduction.

VIII. INFLUENCE OF ISOVECTOR PARTICLE-HOLE FIELDS ON THE MOMENT OF INERTIA IN ISOSPACE

Phenomenological potentials [like the Woods-Saxon potential used here] depend only on the third component of the isospin $I = (N - Z)/A$. It means that essentially no modification of the mean-field is done as a function of excitation energy in a given nucleus. On the contrary, such modifications are automatically included in self-consistent approaches through the changes of the isovector densities.

The presence of a repulsive isovector mean-field provides an alternative [or additional] mechanism to lower the MoI in isospace. Let us illustrate it by using the simple iso-cranked single-particle model discussed in detail in Ref. [14, 15, 16] and including an additionally repulsive two-body interaction $\frac{1}{2}\kappa\hat{\mathbf{t}} \cdot \hat{\mathbf{t}}$ analyzed by Neergård [24]:

$$\hat{H}^\omega = \hat{h}_{sp} - \vec{\omega}\hat{\mathbf{t}} + \frac{1}{2}\kappa\hat{\mathbf{t}} \cdot \hat{\mathbf{t}} \quad (73)$$

Linearization of the Hamiltonian (73) and simultaneous assumption of one-dimensional rotation [say around x -axis, then $\langle\hat{t}_y\rangle = \langle\hat{t}_z\rangle = 0$] leads effectively to a mean-field one-dimensional cranking Hamiltonian:

$$\hat{H}_{MF}^\omega = h_{sp} - (\omega - \kappa\langle\hat{t}_x\rangle)\hat{t}_x \quad (74)$$

with an effective isospin dependent cranking frequency. The role of the isovector field is depicted schematically in fig. 6 where for simplicity the equidistant single particle (s.p.) spectrum $e_i = i\delta e$ for $h_{sp} = \sum_{occ} e_i$ is assumed. It is clearly seen from the figure that isovector field simply shifts crossing frequencies from $\delta e, 3\delta e, 5\delta e, \dots$ to $\delta e + \kappa, 3(\delta e + \kappa), 5(\delta e + \kappa), \dots$. These shifts are marked by dotted lines in fig. 6 since they are in fact instantaneous due to non-collective iso-cranking. The energy dependence $E(T)$ as a function of isospin [$T = T_x \equiv \langle\hat{t}_x\rangle$] reads:

$$E(T) = \frac{1}{2}(\delta e + 2\kappa)T^2 \quad (75)$$

i.e. is analogical to the one derived in [14] but with a reduced MoI-i. Adding standard $\mathbf{t} = 1, t_z = \pm 1$ pairing with $\Delta_n = \Delta_p$ *deformes* the system [see Sec. III] and smoothes out the single-particle, step-like alignment process [*collective iso-rotation*] but does not essentially change the MoI-i as already discussed in [14, 15, 16] [see also Sec. III B]. The *collectivity* effect introduced by the $\mathbf{t} = 1, t_z = \pm 1$ pairing correlations allows for incorporating isospin

fluctuations in a static way through standard cranking condition

$$E = \frac{1}{2}(\delta e + 2\kappa)T^2 \longrightarrow \frac{1}{2}(\delta e + 2\kappa)T(T + 1) \quad (76)$$

and thus restores the linear [Wigner] term. This regime of the model with fixed A , and $T_z = 0$ can be called the *vertical* excitation regime. Changing the iso-cranking generator from \hat{t}_x to \hat{t}_z and limiting ourselves to total particle number conservation A only brings our model to the regime of *horizontal* excitations describing ground states of neighbouring nuclei of the same A . The mathematics used to solve both models is identical. Note, however, that the physics interpretation changes. For example, the physical restrictions for allowed s.p. [or quasiparticle] excitations which were related to iso-signature conservation and time reversal symmetries [14, 15, 16] now can be interpreted in terms of neutron and proton number parities. The cranking frequency measures the difference between neutron and proton Fermi energies.

In the derivation of eqs. (75) the isovector Hartree potential $V_T = \kappa \langle \hat{\mathbf{t}} \rangle \hat{\mathbf{t}}$ was used rather than the two-body interaction. This seems to be consistent with standard potential-like treatment of the isovector terms in phenomenological nuclear potentials. In such a case use of the cranking condition (76) also makes sense. Since for large isospins pn pairing is irrelevant, the inertia parameter [the symmetry energy strength a_{sym}] is fully determined by the mean-level density and κ . The data does not give any signature of enhancement of a_{sym} in $N=Z$ nuclei. Still isoscalar pn -pairing effects may be related to the enhancement of the *linear* term to $\frac{1}{2}a_{sym}T(T + x)$ with $x > 1$.

The symmetry energy strength, and indirectly the value of κ , can be conveniently fitted using the so-called double difference $V_{pn} \approx \frac{\partial^2 B}{\partial N \partial Z}$ formula [41]. The results of a local fit to $N \sim Z$ nuclei involving $Z \geq 10$, $1 \leq T_Z \leq 3$ nuclei [except o-o $T_Z=1$ nuclei] are shown in table II. This fit assumes $E_{sym} \propto (N - Z)^2$ since the linear term vanishes because of the second derivative. However, the strength of the linear term [or the Wigner energy $E_{wig} \propto |N - Z|$] can be fitted separately using the prescription of Ref. [42] which gives information about the local enhancement factor x in the $E_{sym} \sim T(T + x)$ formula.

The obtained strength of the symmetry energy lies in between large scale fits of $E_{sym} \sim T^2$ type [43] and with those assuming $E_{sym} \sim T(T + 1)$. The latest gives:

$$E_{sym} = \left(134.4 - \frac{203.6}{A^{1/3}} \right) \frac{T(T + 1)}{A} \text{ MeV} \approx \frac{1}{2} 160 \frac{T(T + 1)}{A} \text{ MeV} \quad (77)$$

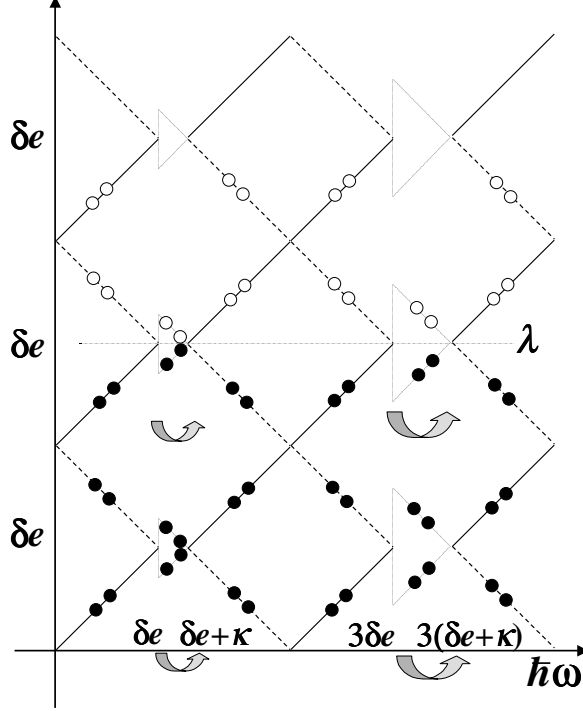


FIG. 6: Single particle routhians representative for a model described by the Hamiltonian (74). Filled circles mark occupied states. Arrows indicate shifts in crossing frequencies due to the isospin dependence introduced via the $\frac{1}{2}\kappa\hat{\mathbf{t}} \cdot \hat{\mathbf{t}}$ interaction.

α	a_W	σ_{n-1}	$2a_{sym}$	σ_{n-1}	$x = a_W/2a_{sym}$
0.95	39	0.196	31	0.106	1.26
1/2	8	0.239	6	0.153	1.33
2/3	14	0.213	11	0.125	1.27
1	47	0.196	38	0.107	1.24

TABLE II: Results of the least-square fit for the Wigner energy strength $E_{wig} = a_W|N - Z|/A^\alpha$ and for the symmetry energy strength $E_{sym} = a_{sym}(N - Z)^2/A^\alpha$. The minimum of the mean standard deviation is reached at $\alpha \sim 0.95$ for both Wigner energy and symmetry energy fits. Results of one-dimensional fits at fixed $\alpha = 1/2; 2/3; 1$ are given for comparison.

for $A \sim 50$ [44]. The data show clear enhancement of the linear term with $x \sim 1.25$ [last column] which is consistent with the early findings of Jänecke [45]. Indeed, it clearly leaves room for isoscalar pairing since isospin fluctuations in the static theory limit gives $x = 1$ by definition.

We have demonstrated recently [46] that the schematic interaction (73) captures many features of realistic effective isovector interactions. In particular, in the Hartree-Fock limit, it gives rise to a *linear term* in the symmetry energy:

$$\frac{1}{2}\kappa\langle\Delta\hat{\mathbf{t}}^2\rangle \xrightarrow{HF} \frac{1}{2}\kappa T. \quad (78)$$

This term is only due to the isovector part of mean HF potential and therefore represents only a fraction of the *linear term*. The mean-level density related contribution

$$\frac{1}{2}\varepsilon\langle\Delta\hat{\mathbf{t}}^2\rangle \quad (79)$$

seems to go beyond the mean-field approximation and its microscopic evaluation requires RPA calculations [24].

Standard *pp/nn* pairing leads to a strong quenching of the linear term (78), as compared to its *sp* estimate [46]. Since for a schematic force κ is fixed the suppression of this term is entirely due to enhancement of the $T/\langle\Delta\hat{\mathbf{t}}^2\rangle$ ratio. This mechanism may also quench the linear term (79). Therefore, from microscopic point of view, one would expect $x < 1$ i.e. below the static estimate. For example, Neergård [24] gives $x \sim 0.8$ for $T = 2$, $A = 48$ nuclei. Let us point out, however, that his estimate is based on a mean-level splitting deduced from Fermi gas model which is unrealistically large as shown in [46].

Moreover, Neergård's model does not take into account $\mathbf{t}=0$ pairing correlations. These correlations, even by entering at the dynamical RPA level, are expected to reduce isospin fluctuations as they in fact do in the static BCS or LN theory, see fig. 7. In conclusion, as expected, the presence of an isovector particle-hole field will definitely reduce isoscalar pairing effects but it does not rule out their existence in $N \sim Z$ nuclei.

IX. SUMMARY AND CONCLUSIONS

We present analytical non-selfconsistent solutions for a model including schematic isovector and isoscalar pairing and discuss in detail the response of the $\mathbf{t}=1$ and $\mathbf{t}=0$ pair-fields

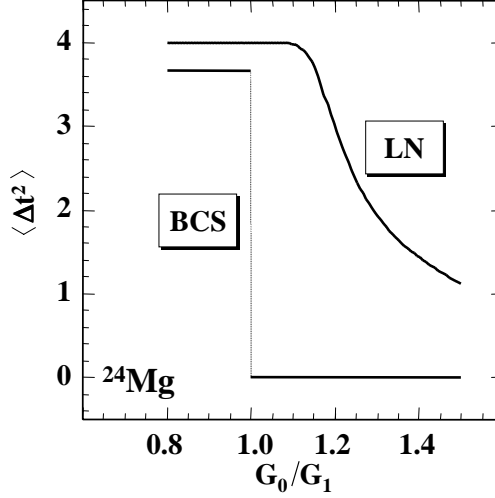


FIG. 7: Dispersion in isospin $\langle \Delta \hat{t}^2 \rangle$ calculated for the LN and BCS models as a function of the ratio of the strengths of isoscalar to isovector pairing forces. Note that $t=0$ pairing strongly quenches $\langle \Delta \hat{t}^2 \rangle$. In particular, in the limit of pure isoscalar pairing (BCS) $\langle \Delta \hat{t}^2 \rangle = 0$.

to rotations in isospace in $N=Z$ nuclei. We are particularly interested in the relations of the gauge angles of the $t=1$ pair-gaps and the position of cranking axis in isospace. These relations decide upon the character of rotation in isospace. In particular, it is shown that within the standard model including nn and pp pair correlations and under the assumption of eq.(12) no tilted solutions are possible. Isorotation is either of collective type $\vec{\Delta} \perp \vec{\omega}$ or non-collective type $\vec{\Delta} \parallel \vec{\omega}$. Since isorotation in $N=Z$ is planar, the pn $t=1$ pair-field always induces collectivity. Again it is shown that the most general solutions are obtained for pure collective cases when the $t=1$ pair-field $[\Delta_{\perp}, \Delta_{pn}]$ is perpendicular to the isocranking axis i.e. when $\vec{\Delta}_{\perp} \perp \vec{\omega}$. Other solutions might be possible only for particular values of the gaps.

We also demonstrate that, within the simple pair-models, mixing of $t=1$ and $t=0$ correlations is essentially forbidden or more precisely restricted to a very special combination of pair gaps. This is due to the different phase structure of eigenvectors among time-reversed states, compare eq. (10) and eq. (43), or alternatively, due to the different transformation properties of $t=1$ and $t=0$ gaps under time-reversal.

Numerical calculations show that this mixing is allowed within the HFB approximation

but only for more elaborate forces [5, 11]. It is also possible for schematic forces, within the LN approximation [2]. In the latter approximation, however, the pn $t=1$ pair gap vanishes which makes it unsuitable to calculate the $T=1$ pair transfer between the $T=1$ o-o ground state and $T=0$ e-e vacuum. Let us point out very clearly that, since $T=1$ o-o ground state and $T=0$ e-e vacuum are according to our interpretation both 0QP states, the $t=1$ pair transfer can in principle become enhanced through $t=1$ pn correlations as it is demonstrated within the BCS approximation in Sec. VI. On the other hand, the different fundamental structure of the $T = 0$ vacua in e-e and o-o nuclei results in a quenching of the $T=0$ pair transfer even in the presence of strong $t=0$ pairing.

The energy difference of $\Delta E_T \equiv E_{T=0} - E_{T=1}$ can be reproduced in a schematic manner by our extended TRS calculations, including a residual pn -force of δ -type. Some discrepancies remain, most likely related to the fact that the $t=0$ pairing is too strong in our calculations due to the lack of particle-hole isovector interaction. Inclusion of such an interaction will result in a weakening of the isoscalar pairing. However, the enhancement in binding energy in $N=Z$ as compared to $N-Z=2$ nuclei seems to leave quite some room for $t=0$ pair correlations as discussed in Sec. VIII. Further work along this line is in progress.

This work was supported by the Göran Gustafsson Foundation, the Swedish Institute, the Polish Committee for Scientific Research (KBN) under Contract No. 5 P03B 014 21, and by the U.S. Department of Energy under Contract Nos. DE-FG02-96ER40963 (University of Tennessee), and DE-AC05-00OR22725 (UT-Battelle).

-
- [1] J. Engel, K. Langanke and P. Vogel, Phys. Lett. **B389**, 211 (1996).
 - [2] W. Satulá and R. Wyss, Phys. Lett. **B393**, 1 (1997).
 - [3] J. Engel, S. Pittel, M. Stoitsov, P. Vogel, and J. Dukelsky, Phys. Rev. **55C**, 1781 (1997).
 - [4] D.J. Dean, S.E. Koonin, K. Langanke, P.B. Radha, Phys. Lett. **B399**, 1 (1997).
 - [5] J. Terasaki, R. Wyss P.-H. Heenen, Phys. Lett. **B437**, 1 (1998).
 - [6] J. Dobes, and S. Pittel, Phys. Rev. **C 57**, 688 (1998).
 - [7] S. Frauendorf and J. Sheikh, Phys. Rev. **C59**, 1400 (1999).
 - [8] S. Frauendorf and J. Sheikh, Nucl. Phys. **A645**, 509 (1999); Phys. Scr. **T88**, 162 (2000).

- [9] K. Kaneko and M. Hasegawa, Phys. Rev. **C60**, 024301 (1999).
- [10] S. Frauendorf and J. Sheikh, Phys. Scr. **T88**, 162 (2000).
- [11] A.L. Goodman, Phys. Rev. **C60**, 014311 (1999).
- [12] G. Röpke, A. Schnell, P. Schuck, and U. Lombardo, Phys. Rev. **C61**, 024306 (2000).
- [13] W. Satulá and R. Wyss, Nucl. Phys. **A676**, 120 (2000).
- [14] W. Satulá and R. Wyss, Phys. Rev. Lett., **86**, 4488 (2001).
- [15] W. Satulá and R. Wyss, Phys. Rev. Lett., **87**, 052504 (2001).
- [16] W. Satulá and R. Wyss, Acta Phys. Pol. **B32**, 2441 (2001).
- [17] H.T. Chen, H. Müther and A. Faessler, Nucl. Phys. **A297**, 445 (1978).
- [18] R.A. Broglia and A. Winther, Frontiers in Physics: A Lecture Note and Reprint Series 52; Reading, Mass. 1981, Benjamin/Cummings.
- [19] D. Bes, R.A. Broglia, O. Hansen, and O. Nathan, Phys. Rep. **34**, 53 (1977).
- [20] D. Bes, O. Civitarese, E.E. Maqueda, and N.N. Scoccola, Phys. Rev. **C61**, 024315 (2000).
- [21] A.O. Macchiavelli *et al.*, Phys. Rev. **C61**, 041303R (2000).
- [22] P. Ring and P. Schuck, *The Nuclear Many Body Problem* (Springer-Verlag, New York, 1980).
- [23] A.L. Goodman, Nucl. Phys. **A186**, 475 (1972).
- [24] K. Neergård, Phys. Lett. **B537**, 287 (2002).
- [25] M. El Nadi, Proc. Phys. Soc., **70**, 62 (1957).
- [26] M. El Nadi and M. El Khishin, Proc. Phys. Soc., **73**, 705 (1959).
- [27] H. C. Newns, Proc. Phys. Soc., **76**, 489 (1960).
- [28] N. K. Glendenning, Nucl. Phys., **29**, 109 (1962).
- [29] S. Yoshida, Nucl. Phys., **33**, 685 (1962).
- [30] P. Fröbrich, Z. Phys. **236**, 153 (1970).
- [31] P. Fröbrich, Phys. Lett. **37B**, 338 (1971).
- [32] D. Fleming, J. Cerny, C. Maples, and N. K. Glendenning, Phys. Rev., **166**, 1012 (1968).
- [33] W.D. Myers and W. Swiatecki, Nucl. Phys. **81**,1 (1966).
- [34] W. Satulá, R. Wyss and P. Magierski, Nucl. Phys. **A578**, 45 (1994).
- [35] R. Firestone *et al.*, *Table of Isotopes*, 8th ed. (Wiley, New York, 1996).
- [36] S.M. Vincent *et al.*, Phys. Lett. **B437**, 264 (1998).
- [37] R. Grzywacz *et al.*, Nucl. Phys. **A682**, 41c (2001).
- [38] D.G. Jenkins *et al.*, Phys. Rev. **C65**, 064307 (2002).

- [39] D. Rudolph *et. al.*, Phys. Rev. Lett. **76**,376 (1996).
- [40] H.J. Krappe, J.R. Nix and A.J. Sierk, Phys. Rev. **C20**, 992 (1979).
- [41] J.-Y. Zhang, R.F. Casten and D.S. Brenner, Phys. Lett. **227B**, 1 (1989).
- [42] W. Satuła, D. Dean, J. Gary, S. Mizutori and W. Nazarewicz, Phys. Lett. **B407**, 103 (1997).
- [43] P. Möller, J.R. Nix, W.D. Myers, W.J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995).
- [44] M. Duflo and A.P. Zucker, Phys. Rev. **C52**, R23 (1995).
- [45] J. Jänecke, Nucl. Phys. **73**, 97 (1965).
- [46] W. Satuła and R. Wyss, Phys. Lett. **B** (2002) submitted.
- [47] At least not for a general case, see the discussion on a similar subject in Sec. IV B.
- [48] Numerical estimate for $N = Z$ nuclei are given in Ref. [13].